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Two-dimensional strain from the orientation of lines in a plane

RENÉE PANOZZO

Abteilung für wissenschaftliche Photographie, Universität Basel, Klingelbergstr. 80, CH-4056 Basel, Switzerland

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Abstract—A fast method for the analysis of two-dimensional strain from the preferred orientation of lines is presented. It is assumed that grain boundary surfaces, or other surfaces, of undeformed polycrystalline rocks have no preferred orientation, that is, that the orientation of surface elements is random. Homogeneous strain of the rock volume is then assumed to produce a preferred orientation of the surface elements. On a two-dimensional section, this appears as preferred orientation of line segments. A simple way of quantifying the preferred orientation of line segments and a general interpretation in terms of two-dimensional strain is shown. This strain analysis, which is based on the change of orientation of surface as a function of strain, is compared to Fry's method that uses the change of relative position of centrepoints as a measure for strain.

INTRODUCTION

ON A THIN section of a polycrystalline rock, mineral grains and matrix are visible as areas; grain boundary surfaces appear as lines or outlines. Although it is by the outlines that we recognize shapes, it is not the outlines themselves that are used for analysis. Commonly, simple geometric approximations are used instead of the actual shape. This approach has yielded a number of powerful techniques of strain analysis (e.g. Ramsay 1967, Shimamoto & Ikeda 1976). These methods will here be referred to as shape methods.

It is the purpose of this paper to show that if one uses the surfaces as they are rather than substituting simplified shapes, one is rewarded by a simple and straightforward method of two-dimensional strain analysis.

Table 1 shows the symbols and definitions used in this paper. It is assumed that the orientation of surfaces in the undeformed rock is random, and that on any section the orientation of the corresponding lines is random too. In other words, for large samples of undeformed rock there should be an equal fraction of lines being oriented within every interval $\Delta\alpha i$ of the angle of orientation α.

On a plane of section, which will here be referred to as the x–y plane, the angle of orientation αi of a straight line is given by the slope of the line with respect to the x-axis. The orientation of a curved line, whose slope changes continuously, is defined at each point by the slope of the tangent. In order to define a 'general' or 'average' orientation of a curved line, the latter is approximated by a set of short straight line segments. Orientations and lengths of the line segments are measured; and a histogram of total length per increment of angle αi is obtained by adding the lengths of all segments for each interval $\Delta\alpha i$. The interval of αi where the total length of line segments has a maximum, that is the mode of the histogram, corresponds to the 'average' orientation of the set of lines, that is to the preferred orientation αp.

In the undeformed rock, orientations of surfaces are assumed to be random. Accordingly, the orientations of lines in a two-dimensional section are random too, and the total length of line segments is constant for all intervals $\Delta\alpha i$, that is, there is no preferred orientation, αp.

For the method presented in this paper to be applicable, one has to assume that a preferred orientation of lines is induced by homogeneous strain, such that on any section the preferred orientation of lines is a function of the respective two-dimensional strain only. Finite strains that are calculated by the method presented here always refer to the state of random orientation of surface as the undeformed state.

PROJECTION OF LINES AND DISTRIBUTION FUNCTIONS

The basic operation of the proposed strain analysis, which will be called the projection method, is to project sets of straight lines or line segments on the x-axis while

Table 1. Symbols used

s	Straight line segment of unit length
m	Number of straight line segments
S	Two-dimensional outline of shape, i.e. closed line: $S = \Sigma s$
αi	Angle of initial orientation of line in x–y plane, measured counterclockwise between line and positive x-axis: $\{0° < \alpha i < 180°\}$
$\Delta\alpha i$	Interval of angle αi
α	Angle of rotation of line or set of lines, measured counterclockwise from positive x-axis: $\{0° < \alpha < 180°\}$
$\Delta\alpha$	Increment of angle α
n	Number of increments $\Delta\alpha$ per 180° rotation: $n = 180°/\Delta\alpha$
x min	Minimum x-coordinate of shape S: $x \min = x \min(\alpha)$
x max	Maximum x-coordinate of shape S: $x \max = x \max(\alpha)$
P	Length of projection of line s: $P = P(\alpha)$
A	Total length of projection of set of lines S: $A(\alpha) = \Sigma P(\alpha)$
B	Simple projection of shape S: $B(\alpha) = x \max(\alpha) - x \min(\alpha)$
α min	Angle α where $A(\alpha)$ or $B(\alpha)$ has a minimum
α max	Angle α where $A(\alpha)$ or $B(\alpha)$ has a maximum
αp	Angle of preferred orientation of set of lines
$h(\alpha i)$	Distribution function of αi
$*$	Convolution
θ	Angle of orientation of strain ellipse, measured between the long axis and the positive x-axis
ψ	Angle of shear

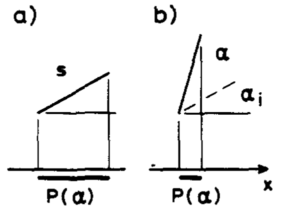

Fig. 1. Projection $P(\alpha)$ of a single straight line, s; initial orientation, $\alpha i = 30°$. (a) Angle of rotation $\alpha = 0°$; (b) $\alpha = 45°$.

the sets are rotated through an angle of 180°. The projection of a single line, s (Fig. 1), is given by

$$P(\alpha) = s\,|\cos(\alpha i + \alpha)|, \tag{1}$$

where the length of the line is s, αi is the initial orientation of the line with respect to the x-axis, and α is the angle of orientation of the line, measured counterclockwise from the positive x-axis. The projection of a set of m lines is given by

$$A(\alpha) = \Sigma P(\alpha), \tag{2}$$

where $A(\alpha)$ depends on the number ot lines in the set, m, their length, s, their initial orientation, αi, and the angle of rotation, α. However, $A(\alpha)$ does not depend on the position of the line segments in the x–y plane.

Unless all lines of the fabric are perfectly parallel, the individual orientations, αi, of various lines will differ. The distribution function $h(\alpha i)$ describes the distribution of the orientation of line segments s. $h(\alpha i)$ represents the density of probability of a line segment being initially oriented at an angle αi. For the discussion of the distribution functions it will be assumed that all line segments are of unit length. If all line segments are parallel, as shown in Fig. 2(a), the distribution function is called monodisperse

$$h(\alpha i)\begin{cases} = 1.00, & \text{if } \alpha i = \alpha p \\ = 0.00, & \text{if } \alpha i \neq \alpha p. \end{cases} \tag{3a}$$

A preferred orientation of lines is represented by a dependence of $h(\alpha i)$ on αi, for example a normal or circular normal distribution function corresponding to a symmetric unimodal distribution, or by a general, even polymodal distribution function if more than one preferred orientation exists in the section. As an example, for a preferred orientation as shown in Fig. 2(b), the normal distribution function is given by

$$h(\alpha i) = 1/(\sqrt{2\pi}\,\sigma)\exp[-(\mu - \alpha i)^2/(2\sigma^2)]. \tag{3b}$$

If the lines are randomly oriented, as shown in Fig. 2(c), the distribution function is called uniform.

$$h(\alpha i) = \text{constant for all } \alpha i. \tag{3c}$$

If the distribution function $h(\alpha i)$ is known, the projection of a set of straight line segments of unit length, $A(\alpha)$, can be calculated; and equation (2) is replaced by the convolution of the projection function of a single line, $P(\alpha)$, and the distribution function, $h(\alpha i)$ (Panozzo 1983)

$$A(\alpha) = P(\alpha) * h(\alpha i). \tag{4a}$$
$$A(\alpha) = \int h(\alpha i) P(\alpha - \alpha i)\,d\alpha i. \tag{4b}$$

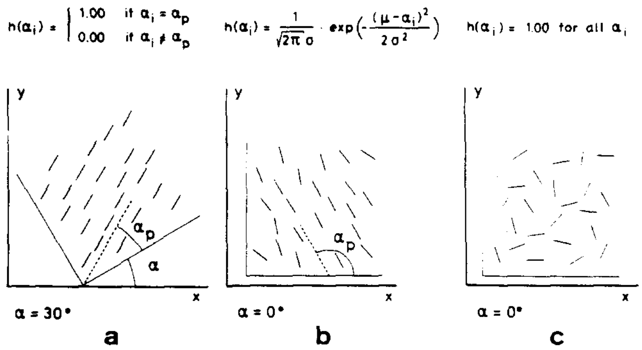

Fig. 2. Sets of straight lines and corresponding distribution functions, $h(\alpha i)$; α = angle of rotation. (a) Parallel lines; monodisperse distribution function; (b) Lines with preferred orientation: normal distribution function; $\mu = 120°$, $\sigma = 10°$; (c) Randomly oriented lines; uniform distribution function. See text for further discussion.

Figure 3 shows projection functions for sets of unit line segments whose initial orientations are described by the monodisperse (M), normal (P), and uniform distribution function (U). $A(\alpha)$ is constant for all angles of rotation, α, if the distribution function $h(\alpha i)$ is uniform, that is, if the orientation of lines is random. $A(\alpha)$ approaches $P(\alpha)$ as the distribution function becomes narrower, for example, as the standard deviation, σ, of the normal distribution function [equation (3b)] decreases. If $h(\alpha i)$ is the monodisperse distribution function, $A(\alpha)$ is essentially equal to $P(\alpha)$.

The 'undeformed state' is here represented by a fabric in which the orientation of lines is random, that is, whose projection $A(\alpha)$ is constant for all α. Homogeneous deformation yields a preferred orientation of lines, that is, $A(\alpha)$ is not constant anymore, but depends on α. In this paper, the function describing $A(\alpha)$ will be given and interpreted in terms of the two-dimensional strain ellipse. In order to do so, projections of ellipses have to be considered first.

If closed lines, that is, shapes are to be projected two types of projection have to be distinguished: $A(\alpha)$, the total projection; $B(\alpha)$, the simple projection or Feret diameter (Fig. 4). $A(\alpha)$, the total projection corresponds to the projection function of a set of straight line segments by which the shape is approximated. These need not be of unit length. $B(\alpha)$, on the other hand, is the difference between the maximum and the minimum

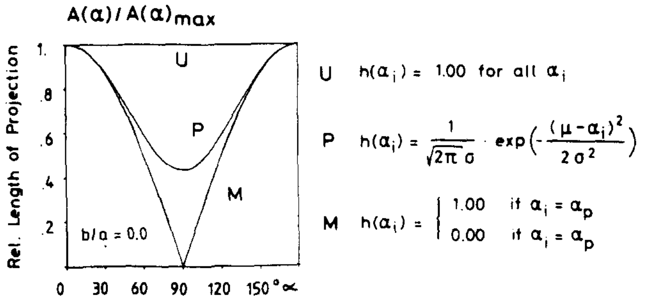

Fig. 3. Relative projection function, $A(\alpha)/A(\alpha)$ max, of sets of straight lines versus angle of rotation α; $\alpha p = 0°$; axial ratio $b/a = 0.00$. M = monodisperse distribution $[h(\alpha i)]$, parallel lines; P = normal distribution $[h(\alpha i)]$, lines with preferred orientation; U = uniform distribution $[h(\alpha i)]$, lines are randomly oriented.

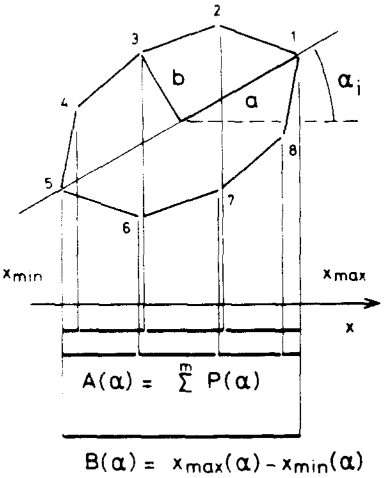

Fig. 4. Projection of ellipse that is approximated by eight straight-line segments; $\alpha i = 30°$; $\alpha = 0°$. Total projection, $A(\alpha)$, and simple projection, $B(\alpha)$, are shown schematically. a and b = axes of ellipse; 1 . . . 8 = digitized points; x min and x max = minimum and maximum x-coordinates of ellipse. See text for further discussion.

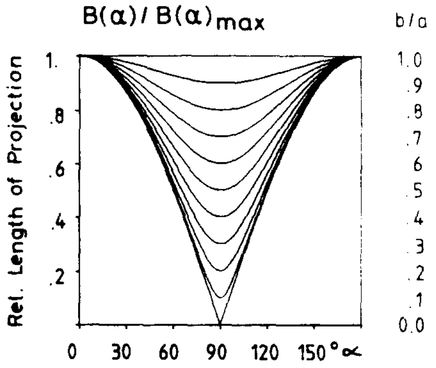

Fig. 5. Simple projection, $B(\alpha)$, of ellipses. Relative length of projection, $B(\alpha)/B(\alpha)$ max, vs angle of rotation, α, for ellipses of various axial ratios b/a; $\alpha i = 0°$.

x-coordinate, x max and x min, of the shape S. In so far as the maximum and minimum x-coordinate of a non-circular shape depend on the orientation of the shape with respect to the x-axis, $B(\alpha)$ is a function of the rotation α.

$$B(\alpha) = x \max(\alpha) - x \min(\alpha). \quad (5)$$

As can be seen from Fig. 4, if the shape is strictly convex the total projection is always twice the simple projection, irrespective of the angle of rotation, α, or the curvature of the outline. The projection of a set of lines by which a convex shape is approximated, $A(\alpha)$, is therefore equal to $2B(\alpha)$. For elliptical shapes the simple projection $B(\alpha)$ is calculated from the axes a and b of the ellipse and the orientation, αi, of the long axis, a, with respect to the x-axis. As the ellipse is rotated in the x–y plane the simple projection changes (Panozzo 1983).

$$B(\alpha) = 2\sqrt{a^2 \cos^2(\alpha i + \alpha) + b^2 \sin^2(\alpha i + \alpha)}. \quad (6)$$

The projection of an ellipse whose long axis is parallel to the x-axis, that is, $\alpha i + \alpha = 0°$, is equal to $2a$ and is the longest possible projection. If the long axis is parallel to the y-axis, that is, $\alpha i + \alpha = 90°$, the projection of the ellipse is equal to $2b$ which is the shortest possible projection. Since ellipses are strictly convex shapes, the following relation holds

$$A(\alpha) \min/A(\alpha) \max = B(\alpha) \min/B(\alpha) \max = b/a. \quad (7)$$

Figure 5 shows the projection functions of ellipses with various axial ratios b/a and an initial orientation parallel to the x-axis, $\alpha i = 0°$. Note that ellipses whose axial ratio b/a equals 0.00, are lines. If $\alpha i > 0°$ or $\alpha i < 0°$, the curves are shifted to the left or right, respectively. The angle of initial orientation, αi, is obtained from the minimum and maximum value of $A(\alpha)$ through the following relations

$$\alpha i = 90° - \alpha \min \quad (8a)$$
$$\alpha i = 180° - \alpha \max. \quad (8b)$$

PROCEDURE

The first step in the analysis by the proposed method is to digitize the lines that represent the surface of interest on a digitizing table. Strings of x–y coordinates, which represent one continuous line each, are transferred to a Fortran programmable computer. If k is the number of points that are digitized on a given line, $k - 1$ is the number of straight line segments by which the line is approximated. The Fortran program SURFOR (Surface orientation) carries out the proposed analysis. The program asks for the size of increments $\Delta\alpha$ of the angle of rotation. $\Delta\alpha$ is chosen according to the desired angular resolution. In the course of the analysis, the lines are rotated through an angle of 180°; therefore the number, n, of angles α at which the total projection of the line fabric is to be evaluated is equal to $180°/\Delta\alpha$. It is unnecessary to rotate the shapes through an angle of 360° because $A(\alpha) = A(\alpha + 180°)$. At each of the n increments of rotation, the total projection $A(\alpha)$ is calculated by summing up the projection of all the straight line segments.

As an example, the shape shown in Fig. 4 is digitized and analyzed by the program SURFOR. The number of strings is 1, the number of digitized coordinate points is 9, as point 1 has to be digitized twice. If instead of one continuous line the 8 individual lines are digitized separately, 8 strings with 2 points each are created. However, the number of line segments (8) and the total projection $A(\alpha)$, are the same in both cases. Figure 6 shows output for analysis with increments of rotation, $\Delta\alpha$, of 10°. Beneath name, date and magnification appears the number of straight line segments used for the analysis. Values of total length of projection, mean length, variance and standard deviation are printed for each increment of rotation. The histogram represents the length of total projection as horizontal bars vs α, the angle of rotation. Figure 6 shows 18 values of $A(\alpha)$ evaluated at α = 10°, 20°, . . . 180°. At α = 60° the projection is minimal, that is the preferred orientation, αp, of the eight lines, which is equivalent to the orientation, αi, of the long axis of the ellipse, is 30° [see equation (8a)].

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****: ELL8.ROS

LENGTH OF STRAIGHT LINE SEGMENTS IN MM

DATE: 8-82
MAGNIFICATION: 1 TIMES

NUMBER OF PROJECTED LINES SEGMENTS: 8
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ANGLE	TOTAL	MEAN	VARIANCE	ST.DEV.
10	724.3877	90.54846	2924.01562	54.07417
20	691.5236	86.44045	1803.02283	42.46202
30	637.6559	79.70699	1247.18213	35.31547
40	564.4207	70.55259	1323.50964	36.38007
50	523.7021	65.21276	1323.27502	36.37685
60	521.7486	65.21857	834.60577	28.88954
70	523.9480	65.24350	1316.75964	36.28718
80	564.1365	70.51707	1325.47253	36.40704
90	637.5422	79.69278	1244.69861	35.28028
100	691.5839	86.44798	1795.76782	42.37650
110	724.6199	90.57749	2912.23950	53.96517
120	778.9187	97.36484	3289.18237	57.35139
130	845.3282	105.66603	2859.47021	53.47401
140	886.0625	110.75781	2577.74292	50.77148
150	899.8844	112.48555	2477.96436	49.77916
160	886.3740	110.79675	2572.16577	50.71653
170	845.9417	105.74271	2848.98999	53.37593
180	779.8156	97.47695	3275.06006	57.22814

HISTOGRAM: TOTAL LENGTH OF PROJECTION VERSUS ANGLE OF ROTATION

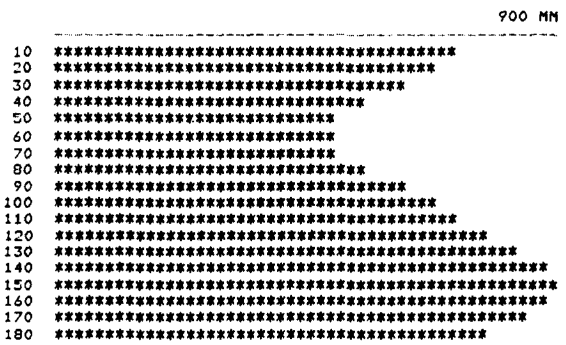

Fig. 6. Sample output of computer program SURFOR. Shape analyzed is ellipse shown in Fig. 4; increment $\Delta\alpha = 10°$. See text for further discussion.

INTERPRETATION OF THE PROJECTION FUNCTION IN TERMS OF TWO-DIMENSIONAL STRAIN

It has to be demonstrated that the axes a and b of the finite strain ellipse and the angle αi between a and the positive x-axis, corresponding to θ (Ramsay 1967), can be derived from the curve $A(\alpha)$ of a deformed fabric.

Consider a pattern of randomly oriented lines (Fig. 7b). These lines can be linked such that they form an isometric polygon. The latter approaches the shape of a circle if the number of line segments is large (Fig. 7a). Owing to the small number (24) of straight-line segments the representations in Fig. 7 are of schematic value only. The total projection $A(\alpha)$ of the circle and of the randomly oriented lines is the same, as is shown in Fig. 7(c). $A(\alpha)$ is constant for all α and is equal to twice the diameter of the circle. If the fabrics shown in Figs. 7(a) & (b) are subjected to the same homogeneous deformation, the fabrics shown in Figs. 7(d) & (e) are obtained. The deformation affects the individual line segments according to their orientation αi, irrespective of their position in the x–y plane. Therefore, the projection function $A(\alpha)$ of the deformed random line pattern and the deformed circle are identical. The projection function is shown in Fig. 7(f). By definition, the shape shown in Fig. 7(d) is the finite strain ellipse. For an ellipse, $A(\alpha)$ is equal to $2B(\alpha)$; and the ratio $A(\alpha)$ min/$A(\alpha)$ max is equal to the axial ratio b/a of the ellipse. Thus the ratio $A(\alpha)$ min/$A(\alpha)$ max of the deformed randomly oriented line pattern is equal to the axial ratio of the finite strain ellipse.

The orientation of the strain ellipse with respect to the reference coordinate system is given by equation (8). In Fig. 7(f), α min $= 180°$ and α max $= 90°$; therefore αp or θ is 90°. The ratio $A(\alpha)$ min/$A(\alpha)$ max is 0.50. The

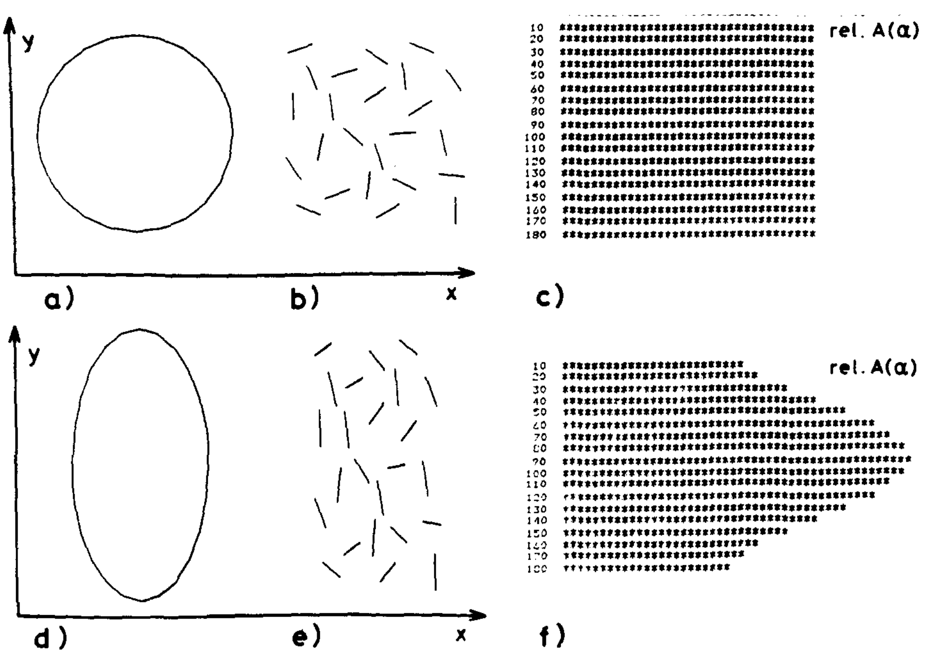

Fig. 7. Correspondence between randomly positioned lines and closed lines. (a) and (b) Undeformed fabric. (c) Computer output. (d) and (e) Deformed fabrics. (f) Computer output. Finite strain axes: $a = 1.4$; $b = 0.7$; angle $\theta = 90°$; increment $\Delta\alpha = 10°$. See text for further discussion.

absolute magnitude of the ellipse axes, b and a, can be determined only if at least one elongation can be measured. Whether this is possible in practice depends on whether the undeformed fabric can be observed and compared to the deformed one.

Deformation of the fabrics shown in Figs. 7(a) or (b) changes both the orientation and the length of the line segments. Thus the individual line segments shown in Figs. 7(d) or (e) are not of unit length anymore. This does not invalidate the analysis because change of length of line segments is taken into account in that the lengths of the individual segments, rather than their number are added. Indeed, if the deformation of the fabrics shown in Figs. 7(a) & (b) consisted of rigid rotation of the lines only, leaving the lengths of lines unaltered, the method proposed here would be inapplicable.

APPLICATION AND DISCUSSION

It is important to define the 'undeformed state' of a rock as it is the reference state with respect to which the finite strain is defined. The 'undeformed state' is often equated to a state of randomness or isotropy, to an absence of preferred orientation or even an absence of fabric. But a general randomness or general preferred orientation does not exist. It has to be specified whether the randomness pertains to position or to orientation. A fabric of isotropic anticlustered positions of centrepoints of particles (Fry 1979) and of random orientations of particle surface need not coincide. Two aspects of the definition of the undeformed state merit special attention: (a) the distinction of random position of particles vs random orientation of particle surface and (b) the distinction of preferred orientation of volumes vs preferred orientation of surface.

Isotropic position of particles versus random orientation of surface

A method of strain analysis based on anticlustered isotropic spatial distributions of centrepoints has been discussed by (Fry 1979). Such spatial distributions exist in nature because particles or grains have finite dimensions, a fact which inhibits centrepoints from lying within a distance smaller than one grain diameter. The concept of anticlustered spatial distributions is here extended to cases where the particles have no volume at all but are small planar surface elements.

Deformation changes the spatial relationship between centrepoints of particles or surface elements by shortening the distances parallel to the direction of compression and by lengthening them parallel to the direction of extension. If one assumes that the change of relative position of centrepoints reflects the state of deformation, the latter can be determined by using the method described in Fry's paper. In many cases, this type of bulk deformation is also reflected by a change of shape, for example, flattening of the particles contained in the rock volume. However, if the viscosity of particles is much

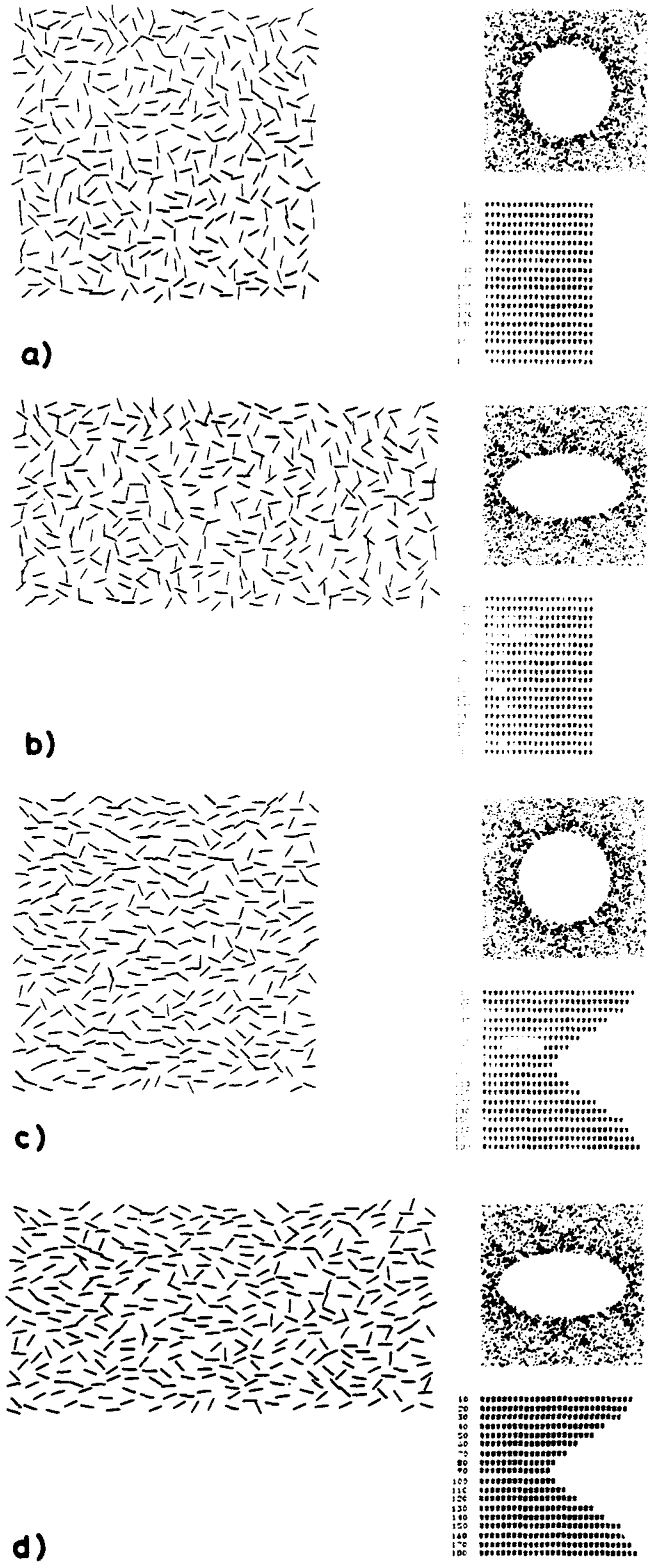

Fig. 8. Comparison of projection method with method after Fry (1979). Analysed line pattern (left); plot of centrepoints (top right); output of program SURFOR (bottom right). (a) Isotropic position, random orientation. (b) Anisotropic position, random orientation. (c) Isotropic position, preferred orientation. (d) Anisotropic position, preferred orientation. See text for further discussion.

higher than the viscosity of the matrix, the particles change their relative position with respect to one another without appreciable change in shape. In such cases the strain determined from the deformation of the particles does not reflect the bulk strain of the rock.

Figure 8 shows four patterns of short lines (e.g. cracks) in the x–y plane. To the right of each the results of two different methods of strain analysis are presented: Fry's method (top) and the projection method described here (bottom). If the isotropic distribution of centrepoints is used to define the undeformed state, Figs. 8(a) & (c) appear to be undeformed and Figs. 8(b) & (d) appear to be strained, with the finite strain ellipse having an axial ratio b/a of 0.50.

Based on the concept of random orientation of lines, Fig. 8 is interpreted differently. If the preferred orientation of lines is assumed to represent bulk strain, Figs. 8(a) & (b) appear undeformed whereas Figs. 8(c) & (d) display a finite strain, where b/a is equal to 0.50. Reconstruction of the undeformed state can be attempted by reversing the deformation. 'Unstraining' Fig. 8(d) by inverting the strain obtained from Fry's method or the one obtained from the projection method produces an 'undeformed' state equivalent to Fig. 8(a). But if the strains of Figs. 8(b) & (c) as determined by the two different methods are reversed, the resulting 'unstrained' patterns are not identical nor do they correspond to the fabric shown in Fig. 8(a).

This demonstrates clearly the necessity of defining the undeformed state in terms of either isotropic position or random orientation of surface. Although the two often coincide, this is not necessarily so; values of strain that are obtained by the two different methods may be different, reflecting an initial fabric or strain partitioning during deformation. If strain measurements of both methods coincide one can be more confident that the obtained values reflect bulk strain. In how far one of the two conditions, isotropy of position or random orientation of surface, is at all realized in nature is yet another question which will not be addressed here.

Preferred orientation of surface vs preferred orientation of shape

In any strain analysis it is assumed that the deformation of rocks is homogeneous within the volume of interest, and that the geometrical elements that are affected by it have existed before deformation started. Elements that are newly created in the course of deformation, for example, pressure-solution surfaces, grain boundaries and cracks, have to be excluded from the analysis.

In absence of contradictory evidence it is often assumed that grains or particles of the undeformed rock are isometric. The grain shapes of the deformed rocks are then approximated by ellipses, and the grain shapes of the undeformed rock are assumed to be circles. Alternatively, the undeformed state may be defined by non-isometric shapes whose long axes are randomly oriented. In general, both of these definitions of the undeformed state are equivalent to a state of random orientation of surface. Therefore, if the state of random orientation of surface and the state of isometric grain shapes or random orientation of non-isometric particles coincide, the projection method described here should yield the same analytical results as any shape method.

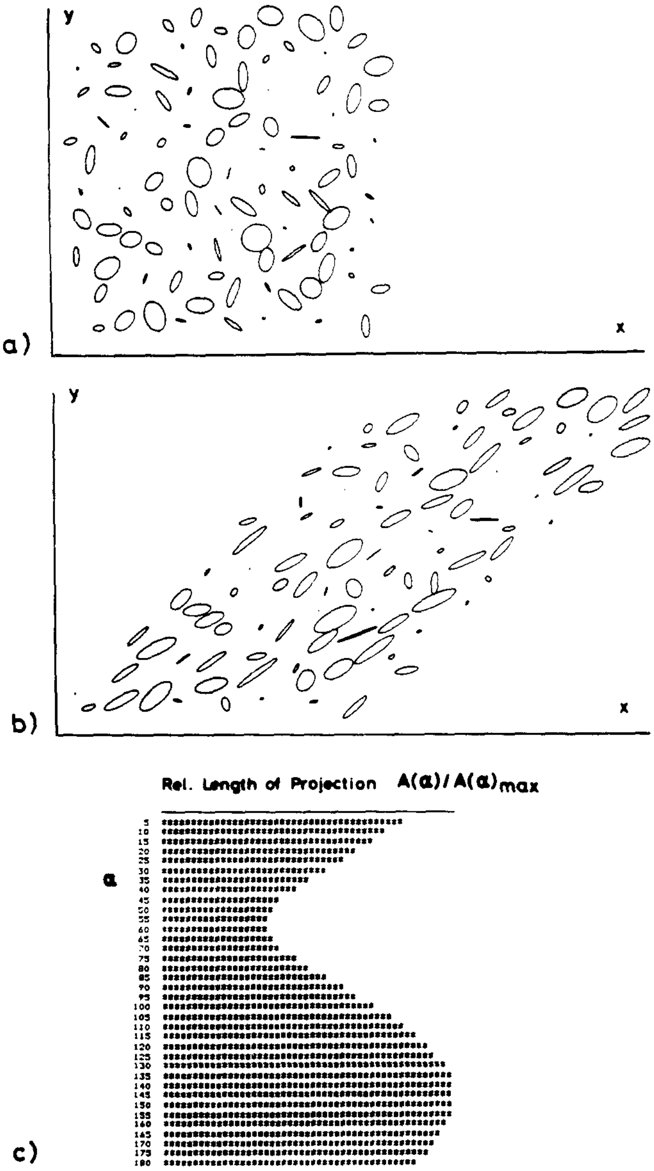

Fig. 9. Deformation of a set of ellipses by simple shear. (a) Random orientation of long axes; normal distribution of axial ratios: $b/a = 0.50 \mp 0.20$; uniform distribution of lengths of long axes: $\{0.00 < a < 1.00\}$; anticlustered isotropic distribution of centrepoints. (b) Deformed fabric; $\psi = 45°$. (c) Projection function of deformed ellipses; $\Delta\alpha = 10°$. See text for further discussion.

There are cases where the alternative use of the proposed method for strain analysis may be favoured.

(1) Strain analysis by any one of the shape methods requires ellipses to be fitted to the shapes. If the outlines are only remotely elliptical the fit of ellipses may be difficult and/or biased. The projection method requires no such fit.

(2) If the strain is small, deviation from sphericity is small and a large number of measurements of ellipse axes is needed. By digitizing the outlines as is proposed here, a large data base can be assembled in a short time.

(3) The analysis of strain from a pattern of deformed ellipses is made easy when compared with shape methods that make use of axial ratios and orientation of long axes (e.g. Ramsay 1967, Shimamoto & Ikeda 1976). Figure 9(a) shows a set of ellipses of different size and axial ratio. They were created using a random number

generator: distribution of angles αi and of lengths is uniform, distribution of axial ratios is normal with the average axial ratio being 0.50 ∓ 20. When deformed by simple shear, the pattern shown in Fig. 9(b) is created. Digitization and evaluation of the fabric take about 20–30 minutes after which the computer output shown in Fig. 9(c) is obtained. $A(\alpha)$ min/$A(\alpha)$ max = 2300 mm/ 6200 mm = 0.37. The minimum of $A(\alpha)$ is at 55–60°; from this orientation αp of the long axis of the strain ellipse is inferred to be 30–35°. Following equations (3-67) and (3-70) given by (Ramsay 1967), and using ψ = 45°, the axial ratio of the strain ellipse comes out to be 0.38, and the angle θ, which corresponds to αp, comes out to be 32°. Better coincidence of the results of SURFOR and the theoretical values can be achieved if smaller increments, $\Delta\alpha$, of rotation are chosen for SURFOR.

(4) The shape methods are restricted to elliptical shapes. The projection method on the other hand is generally applicable whether shapes are elliptical or not. Since only the orientations of line segments are considered it does not even matter whether the lines are open or closed or whether they are individual lines such as cracks or a connected network such as grain boundaries in crystalline rocks. Using this method, strain analysis is possible even if the long dimension of particles is larger than the field of observation on a thin section.

SUMMARY AND CONCLUSIONS

A new method for the analysis of two-dimensional strain has been introduced. The method is based on approximating, that is, digitizing outlines of shapes or other lines by sets of small straight lines. The latter are projected on the x-axis while being rotated through an angle of 180°. From the projection function $A(\alpha)$ (= total length of projection vs angle of rotation) the axes of the two-dimensional finite strain ellipse and their orientation with respect to a reference x–y coordinate system is derived very easily. The method is sensitive to the orientation of lines but not to their position in the x–y plane. The method is therefore complementary to Fry's (1979) method which uses the anisotropy of an anticlustered spatial distribution of centrepoints as a measure of strain.

By using digitized data and the Fortran program SURFOR, strain analysis is achieved in a short time. The proposed method is general in that it applies to all lines or outlines of shapes. Finite strains can be derived from all sets of shapes or line systems, including, for example sets of different ellipses, provided their surface was initially randomly oriented.

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